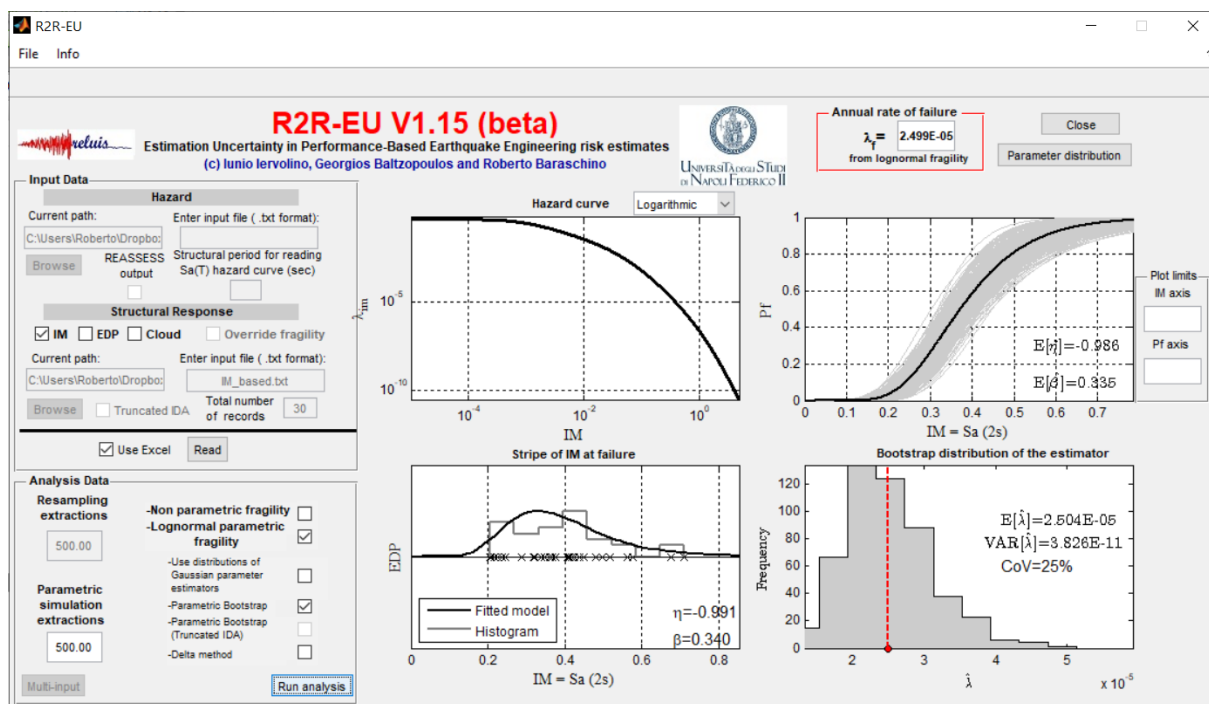# R2R-EU V1.15

Estimation uncertainty in performance-based earthquake engineering risk assessment

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# USER'S MANUAL

JULY 2020

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Installation

Download the following files

- *R2R-EU setup file*

- *MATLAB MCR v8.1 x64*

STEP 1: Installing *MATLAB Component Runtime (MCR) v 8.1 x64*

Because R2R-EU was developed in MATLAB programming environment, prior to running R2R-EU, it is necessary to download and install the MCR v8.1 x64 component (which is also freely distributable and can be downloaded at http://it.mathworks.com/products/compiler/mcr/). After downloading, launch the MCRinstaller.exe and follow the instructions of the installation procedure. Note that there is no problem associated with having MATLAB and the MCR installed simultaneously (even if multiple versions of each are installed).

STEP 2: Installing R2R-EU
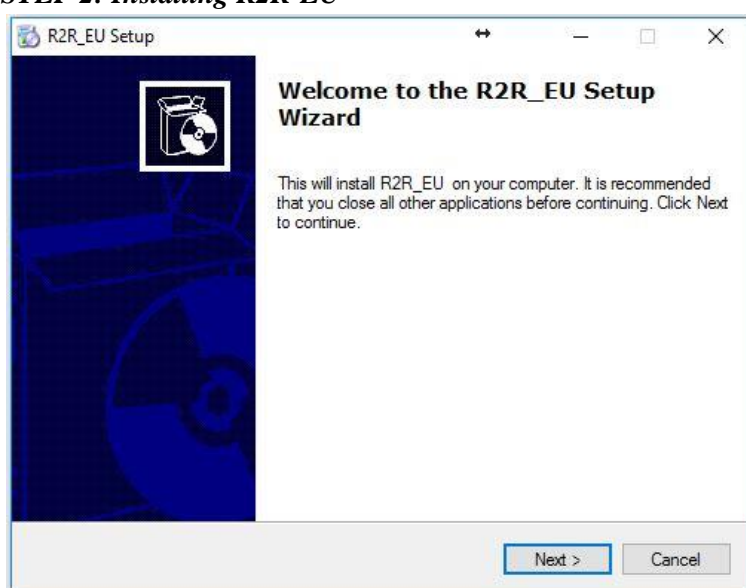
In order to install R2R-EU itself, please launch the installer (R2R-EU_setup.exe) and follow the instructions appearing on the screen (Figure 1). After the installation is completed, it will be possible to run the software (the main executable file, found in the installation folder, is R2R-EU.exe).

Figure 1: R2R-EU installer

NOTE: If it should happen that R2R-EU fails to interact successfully with Excel, that could be a consequence of the program being denied the necessary permissions by the system. This occasional problem can be remedied by simply running the program with administrator privileges (… **Run as administrator**).

Estimation uncertainty in fragility functions and seismic risk: overview of key concepts

The objective of this introductory primer, which is largely based on a technical note published by the authors [1], is to recall some key concepts for fitting fragility functions and quantifying estimation uncertainty in seismic risk and to familiarize the reader with the methods implemented in the R2R-EU tool. The starting point of this discussion is structure-specific seismic risk assessment according to the performance-based earthquake engineering paradigm (PBEE, [2]), which involves the evaluation of the rate of earthquakes, λ_f, causing failure of the structure. This rate can be calculated according to Equation (1):

$$\lambda_f = \int_{IM} P\big[f \,|\, IM = im\big] \cdot \big| d\lambda_{im} \big|, \tag{1}$$

where the term $P\big[f \,|\, IM = im\big]$, that is, the probability of failure for a given seismic intensity, is typically named *structural fragility*. It is a function providing the conditional probability of failure given a certain value im of a ground motion intensity measure IM. On the other hand, the term $\big| d\lambda_{im} \big|$ depends on the derivative of the *hazard curve* for the site of interest. A hazard curve provides the annual rate, λ_{im}, of exceeding each im value at a given site.

The R2R-EU tool deals with the definition of structure-specific seismic fragility and with the quantification of that part of estimation uncertainty in the failure rate, that can be attributed to the record-to-record variability of structural response. The latter can be provided by one of the methods mentioned in the introduction; i.e., IDA, MSA and cloud. IDA involves progressively scaling each ground motion in a set, so as to cover a broad range of IM levels, and running dynamic analysis, ideally until the numerical model experiences instability that can be interpreted as side-sway structural collapse [3]. A measure of structural response, often termed an *engineering demand parameter* or EDP, is being continuously registered at each IM level. The output of this analysis is a set of EDP-IM curves (equal in number to the number of ground motion records used, Figure 2a) that can be rendered quasi-continuous by employing some type of interpolation [4]. On the other hand, MSA involves the use of different sets of – ideally unscaled – accelerograms per IM level, chosen to represent the most likely seismic scenario causing that level of shaking at the construction site [5]. The output of MSA is a set of EDP responses at fixed IM values (Figure 2b). Cloud analysis (Figure 2c) uses a set of unscaled accelerograms to perform dynamic analysis so that at each record represents a single IM value and corresponds to a single EDP response (e.g., [6]).
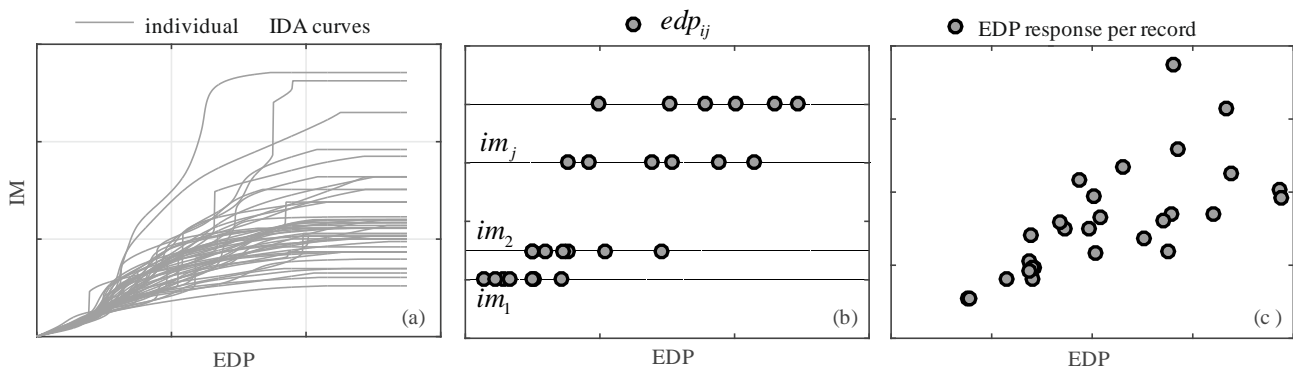
Figure 2: Example of IDA curves of an inelastic frame structure (a); EDP responses of a non-linear structure at four IM levels obtained via MSA (b); EDP-IM response for the same structure obtained via cloud analysis (c).

Fitting fragility functions based on dynamic analysis

Given the output of dynamic analysis, the strategy for analytically evaluating a fragility function often branches into one of two approaches: the IM-based approach and the EDP-based approach; IM-based fragility estimation is suitable within the IDA framework, while EDP-based can be applied in both IDA and MSA settings. In both cases, it is assumed that a threshold EDP, edp_f, can be defined, so that its exceedance will be tantamount to failure, that is, $P\big[f \,|\, IM = im\big] = P\big[EDP > edp_f \,|\, IM = im\big]$. In the IM-based approach, a new random variable

(RV) needs to be introduced: the IM-value causing failure, denoted as IM_f, which is, in principle, different for each record. After the analysis has been concluded and the IDA curves become available, a sample of the RV can be obtained by finding the intersection $im_{f,i}$ between the vertical line passing through edp_f and the i-th IDA curve, $i = \{1, 2, ..., n\}$ (Figure 3a). The fragility function may then be considered as the probability of IM_f being equal or lower than the level of seismic intensity possibly occurring at the site: i.e., $P\left[f | IM = im\right] = P\left[IM_f \leq im\right]$. It is also possible to assign a parametric model to the distribution of IM_f and a typical choice is the lognormal model, which is completely defined by mean η and standard deviation β of the logarithm of IM_f. In that case, fragility can be expressed using the standard Gaussian function, $\Phi(\cdot)$, according to Equation (2):

$$P\left[f | IM = im\right] = P\left[IM_f \leq im\right] = \Phi\left(\frac{\ln(im) - \hat{\eta}}{\hat{\beta}}\right). \tag{2}$$

The two parameters $\{\eta, \beta\}$ are generally unknown and one way to obtain estimates of these parameters, $\{\hat{\eta}, \hat{\beta}\}$, is by using the sample of responses resulting from IDA according to Equation (3):

$$\begin{cases} \hat{\eta} = \frac{1}{n} \cdot \sum_{i=1}^{n} \ln\left(im_{f,i}\right) \\ \hat{\beta} = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^{n} \left[\ln\left(im_{f,i}\right) - \hat{\eta}\right]^2} \end{cases}, \tag{3}$$

where n represents the number of IDA curves and is therefore equal to the number of records used and $im_{f,i}$ is the intensity that one needs to scale the i-th record (out of n in total), in order to cause failure of the structure.
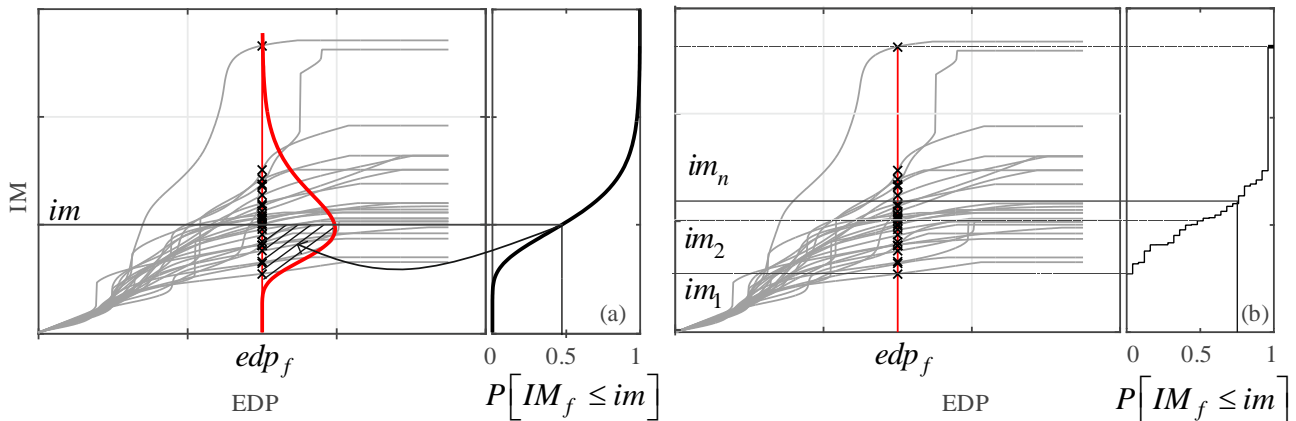
Figure 3: Fragility assessment using IDA curves assuming either a parametric (lognormal) probabilistic model for the distribution of IM_f (a) or a non-parametric representation (b).

Of course, it is not necessary to assume a parametric model for IM-based fragility; in fact, a non-parametric representation can be obtained directly from the sample of IM_f values, according to Equation (4):

$$P\left[f | IM = im\right] = \frac{1}{n} \cdot \sum_{i=1}^{n} I_{\left(im_{f,i} \leq im\right)}, \tag{4}$$

where $I_{\left(im_{f,i} \leq im\right)}$ is an indicator function that returns 1 if $im_{f,i} \leq im$ or 0 if $im_{f,i} > im$ (Figure 3b). The use of estimation uncertainty as a means for determining the number of records to use in IM-based fragility derivation was explored in [7].

Structural fragility can also be expressed as the complementary cumulative distribution function of IM_f, by following an EDP-based approach. In fact, the EDP-based method works both for IDA and MSA; in this case, EDP responses are obtained at discrete (fixed) IM levels. When these EPD responses are plotted against the corresponding IMs, they are arranged in horizontal stripes (e.g., Figure 2b), one for each level of shaking intensity considered. By counting the fraction of records in each stripe that cause the exceedance of the limit state threshold, edp_f, the estimation of the fragility parameters $\{\hat{\eta}, \hat{\beta}\}$ can be carried out via maximum likelihood, according to Equation (5), which is from [8]:

$$\{\hat{\eta}, \hat{\beta}\} = \arg\max_{\eta, \beta} \left[\sum_{j=1}^{u} \left(\ln\binom{n}{q_j} + q_j \cdot \ln\left\{ \Phi\left[\frac{\ln(im_j) - \eta}{\beta} \right] \right\} + (n - q_j) \cdot \ln\left\{ 1 - \Phi\left[\frac{\ln(im_j) - \eta}{\beta} \right] \right\} \right) \right], \quad (5)$$

where u is the number of IM levels considered (i.e., the number of stripes, each stripe containing responses from n records), and $q_j = \sum_{i=1}^{n} I_{(edp_{ij} > edp_f)}$ is the number of failures observed at the stripe corresponding to $IM = im_j$ (Figure 4a), when edp_{ij}, $i = \{1,..,n\}$, $j = \{1,..,u\}$ represents the single structural response recorded at the i-th record of the j-th stripe. In this formulation, cases of non-convergent analysis (referred to as *collapse* cases) due to the numerical model coming too close to highly-nonlinear behaviour associated with incipient instability, say c_j in number, are also counted in q_j and are therefore accounted for, despite the potential lack of a credible EDP value [3]. At this point it should be noted that, in cases where the observed numbers of failure q_j remain excessively low over all stripes considered, the maximum likelihood estimates implied in Equation (5) can suffer from numerical problems. One viable alternative for considering fragility, consistent with the EDP-based approach, is the three-parameter-per-intensity model adopted by Shome and Cornell [3], given by Equations (6):

$$\begin{cases} P\left[f \mid IM = im_j\right] = P\left[\bar{C} \mid IM = im_j\right] \cdot P\left[EDP > edp_f \mid \bar{C}, IM = im_j\right] + P\left[C \mid IM = im_j\right] = \\ = \left(1 - \frac{c_j}{n}\right) \cdot \left\{ 1 - \Phi\left[\frac{\ln(edp_f) - \hat{\eta}_{\ln(EDP_j)}}{\hat{\beta}_{\ln(EDP_j)}} \right] \right\} + \frac{c_j}{n} \\ \hat{\eta}_{\ln(EDP_j)} = (n - c_j)^{-1} \cdot \sum_{i=1}^{n-c_j} \ln(edp_{ij}) \\ \hat{\beta}_{\ln(EDP_j)} = \sqrt{(n - c_j - 1)^{-1} \cdot \sum_{i=1}^{n-c_j} \left[\ln(edp_{ij}) - \hat{\eta}_{\ln(EDP_j)} \right]^2} \end{cases} \quad (6),$$

Where $P\left[C \mid IM = im_j\right]$ is the probability of collapse, $P\left[\bar{C} \mid IM = im_j\right]$ is its complement, i.e., the probability of non-collapse, c_j is the number of observed collapse cases at the j-th stripe, according to the previous definition, and $\hat{\eta}_{\ln(EDP_j)}$, $\hat{\beta}_{\ln(EDP_j)}$ are the mean and the standard deviation of the logarithm of EDP, at $IM = im_j$, provided by structural analysis and not affected by numerical instability (no-collapse cases). Note that this approach provides fragility at the discrete intensities $IM = im_j$, rather than as a continuous function of IM. However, it is also possible to fit a continuous parametric model for the probability of collapse, as an alternative to the empirical fractile c_j/n, by means of logistic regression [9] according to Eq. (7):

$$P\left[C \mid IM = im_j\right] = \frac{1}{1 + e^{-(\alpha_0 + \alpha_1 \cdot im_j)}} \quad (7)$$
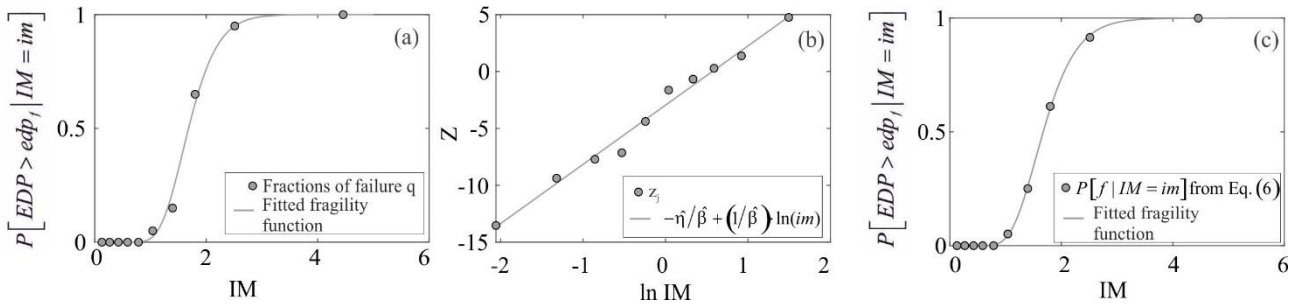
Figure 4: Maximum likelihood fit of a lognormal fragility function to the results of MSA (a); fit of lognormal fragility on normal probability paper (b); fit of lognormal fragility by means of non-linear least squares (c).

Another alternative procedure is to obtain estimates of the parameters $\{\hat{\eta}, \hat{\beta}\}$ by means of least squares fitting, which can be done in one of two ways. One method consists of plotting the failure probabilities per stripe, $P\left[f \mid IM = im_j\right]$ obtained from Equation (6), on normal probability paper and estimating the parameters via least squares fitting of a line. For this procedure, the values of the standard normal variable, Z, corresponding to the failure probabilities are calculated as $z_j = \Phi^{-1}\left(P\left[f \mid IM = im_j\right]\right)$, for which it can be assumed that, on a *normal probability paper* [10], a linear relationship of the form $Z = -\hat{\eta}/\hat{\beta} + \left(1/\hat{\beta}\right) \cdot \ln(im)$ should hold; e.g., Figure 4b. In this case, the least squares estimate for $\{\hat{\eta}, \hat{\beta}\}$ is known to be the one given by Equations (9):

$$
\begin{cases}
\hat{\beta} = \dfrac{\sum_{j=1}^{u}\left(z_j - \overline{z}\right)^2}{\sum_{j=1}^{p}\left(z_j - \overline{z}\right) \cdot \left[\ln\left(im_j\right) - \overline{\ln\left(im\right)}\right]} \\[4ex]
\hat{\eta} = \overline{\ln\left(im\right)} - \dfrac{\overline{z}}{\hat{\beta}} \\[3ex]
\overline{z} = u^{-1} \cdot \sum_{j=1}^{u} z_j \\[3ex]
\overline{\ln\left(im\right)} = u^{-1} \cdot \sum_{j=1}^{u} \ln\left(im_j\right)
\end{cases}
\tag{8},
$$

which are derived directly from the normal equations of the least-squares problem; e.g., [11]. It should be noted that if the probability of failure decreases with increasing seismic intensity, input data may be ill-conditioned and the fragility fitting may be not reliable.

The second method entails fitting a fragility function by minimizing the sum of squared errors according to Equation (10):

$$
\{\hat{\eta}, \hat{\beta}\} = \arg \min_{\eta, \beta} \left(\sum_{j=1}^{u} \left\{ \left(1 - \frac{c_j}{n}\right) \cdot \left\{ 1 - \Phi\left[\frac{\ln\left(edp_f\right) - \hat{\eta}_{\ln\left(EDP_j\right)}}{\hat{\beta}_{\ln\left(EDP_j\right)}} \right] \right\} + \frac{c_j}{n} - \Phi\left(\frac{\ln\left(im_j\right) - \eta}{\beta} \right) \right\}^2 \right)
\tag{9},
$$

which is akin to performing a non-linear least squares regression, Figure 4c.

Quantification of estimation uncertainty in R2R-EU

All of the aforementioned approaches for estimating a fragility function (which may entail assigning a parametric model or not; e.g., Figure 3) have been implemented in R2R-EU. They also allow to quantify the estimation uncertainty in the failure rate, used in conjunction with methodologies of statistical inference. These methodologies, summarized in Table 1 and partly discussed in [12], but also outlined in the following paragraphs, are: parametric or non-parametric resampling plans (generally belonging to the bootstrap family), the application of known results for the distribution of the estimators of the lognormal parameters, and the

delta method, which is based on Taylor series expansion of either the risk integral or the formula from Cornell's seismic reliability method.

Approach	Dynamic analysis	Fragility model	Method for quantifying estimation uncertainty
IM-based	IDA	Lognormal	Parametric bootstrap, probability theory, Delta method
		Non-Parametric	Bootstrap (resampling)
EDP-based	IDA/MSA	Lognormal	Parametric bootstrap
		Shome & Cornell three parameter model per intensity [3]	Bootstrap (resampling)
		Normal probability paper	Bootstrap (resampling)
		Least squares fit	Bootstrap (resampling)
	Cloud	-	Delta method for Cornell's method

Table 1: Methodologies implemented in the R2R-EU tool for the quantification of record-to-record estimation uncertainty in PBEE.

Estimators of the Gaussian distribution's parameters

If structural fragility is assumed lognormal, the estimators of the parameters logarithmic mean $\left(\hat{\eta}\right)$ and variance $\left(\hat{\beta}^2\right)$, obtained according to Equation (3), have known distributions (e.g., [13]). The estimator $\hat{\eta}$ is distributed as a Gaussian with mean and variance equal to η and β^2/n respectively (but assumed equal to $\hat{\eta}$ and $\hat{\beta}^2/n$; i.e., η and β^2 are substituted by the available point estimates), while the estimator $\hat{\beta}^2 \cdot (n-1)/\beta^2$ is *chi-squared* distributed with $n-1$ degrees of freedom. Since the failure rate is a function of these two stochastically independent RVs, shown here as Equation (11):

$$\hat{\lambda}_f\left(\hat{\eta},\hat{\beta}\right) = \int_{IM} \Phi\left[\frac{\ln(im)-\hat{\eta}}{\hat{\beta}}\right] \cdot \left|d\lambda_{im}\right|, \tag{10}$$

it follows that the mean and variance of $\hat{\lambda}_f$ can be evaluated (in R2R-EU) using their known densities.

Bootstrap

The bootstrap is a statistical inference process, which is based on taking an original data set and generating, so-called, *bootstrap samples* by resampling the original data with replacement. The bootstrap samples have the same size as the original. This resampling process is implemented in R2R-EU for three cases: non-parametric IM-based fragility derived from IDA, EDP-based fragility using the three-parameter model of Equation (6) and EDP-based fragility with parameter estimation via the normal probability paper procedure.

Resampling for IM-based fragility

In the case of IM-based non-parametric fragility, the bootstrap implementation takes the original *n*-size sample of IM_f realizations already available from IDA, $\left\{im_{f,1}, im_{f,2}, .., im_{f,n}\right\}$, and generates an arbitrary number, *m*, of bootstrap samples $\left\{im^*_{f,1k}, im^*_{f,2k}, .., im^*_{f,nk}\right\}$, where $k=\{1,..,m\}$. Subsequently, a Monte Carlo simulation is performed, where, for each bootstrap sample, Equation (1) is used to compute a *bootstrap replication* of the failure rate, $\hat{\lambda}^*_{f,k}$. Then, the mean and variance of the failure rate estimator (denoted via the operators $E[\cdot]$ and $VAR[\cdot]$, respectively) are evaluated using the simulations results according to Equation (12):

$$\begin{cases} E\left[\hat{\lambda}_f\right] = \frac{1}{m} \cdot \sum_{k=1}^{m} \hat{\lambda}_{f,k}^* \\ VAR\left[\hat{\lambda}_f\right] = \frac{1}{m-1} \cdot \sum_{k=1}^{m} \left(\hat{\lambda}_{f,k}^* - E\left[\hat{\lambda}_f\right]\right)^2 \end{cases} . \tag{11}$$

Resampling for EDP-based fragility

In the case of EDP-based fragility, the bootstrap process starts from a set of $n \times u$ EDP responses, available from either MSA or IDA and denoted as previously by edp_{ij}, obtained from n records $\left(i = \{1,..,n\}\right)$ at each one of IM levels $\left(j = \{1,..,u\}\right)$, denoted as $\{im_1, im_2,.., im_u\}$. As a first step, the EDP responses at each IM level (stripe), are resampled with replacement m times, resulting in new sets of responses at the j-th stripe (i.e., bootstrap samples) $\{edp_{1j,k}^*, edp_{2j,k}^*,.., edp_{nj,k}^*\}$, $k = \{1,..,m\}$. Subsequently, at each and every j-th, $j = \{1,..,u\}$, stripe of the k-th bootstrap sample, $k = \{1,..,m\}$, the responses $c_{j,k}^*$, corresponding to collapse cases, are identified, and the probabilities of failure, $P_k^*\left[f \mid IM = im_j\right]$, are calculated according to Equation (6).

In the case of the three-parameter model, the k-th bootstrap replication of the failure rate, $\hat{\lambda}_{f,k}^*$ is evaluated according to a discretized version of Equation (1), reported here as Equation (13):

$$\hat{\lambda}_{f,k}^* = \sum_{j=1}^{u} P_k^*\left[f \mid IM = im_j\right] \cdot \left|\Delta\lambda_{im_j}\right| \tag{12},$$

where $\Delta\lambda_{im_j} = \left(\lambda_{im_j} - \lambda_{im_{j-1}}\right)$. This distinction is made due to the fact that the probability of failure is only available at specific, discrete IM levels. Finally, after having obtained all bootstrap replications $\hat{\lambda}_{f,k}^*$, the mean and variance of the failure rate estimator are again evaluated according to Equation (11).
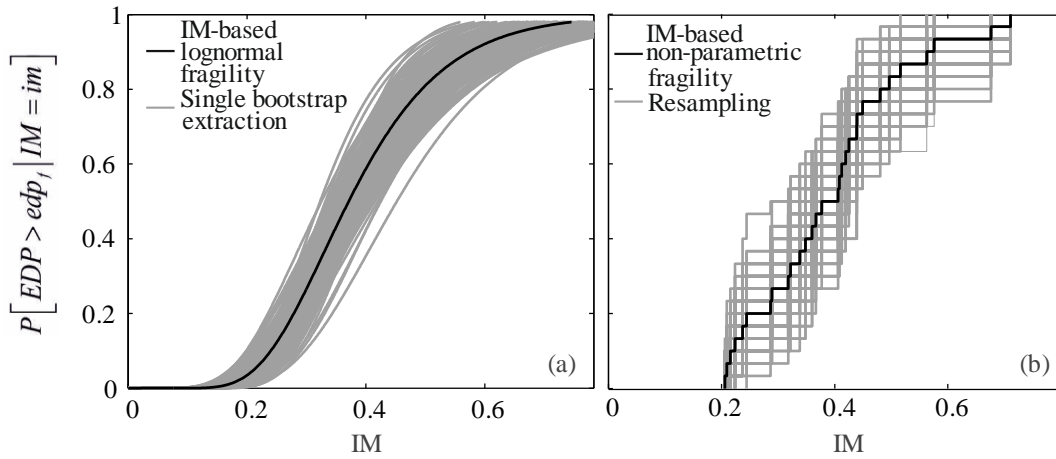
In the case of EDP-based lognormal fragility whose parameters are estimated via linear fit on normal probability paper, new parameter estimates $\{\hat{\eta}_k^*, \hat{\beta}_k^*\}$, $k = \{1,..,m\}$, are obtained via Equation (8) for each bootstrap sample. Then bootstrap replications $\hat{\lambda}_{f,k}^*$ are calculated by substituting the parameters $\{\hat{\eta}_k^*, \hat{\beta}_k^*\}$ into Equations (2) and (1). Finally, Equation (11) is used to obtain the statistics of the failure rate estimator, same as before.

Parametric bootstrap

When a parametric model is assumed for the fragility function, the mean and variance of the failure rate's estimator can be inferred via a parametric version of the bootstrap. In the parametric version, bootstrap samples can be extracted directly from the assumed fragility model, rather than by means of resampling the original dataset. In R2R-EU this is implemented for both cases of IM- and EDP-based lognormal fragility (in the EDP-based case, when parameter estimation occurs via maximum likelihood).

Parametric bootstrap for IM-based fragility

In the IM-based case, the n values of IM_f obtained from IDA, $\{im_{f,1}, im_{f,2},.., im_{f,n}\}$, are used to derive the reference lognormal fragility parameters $\{\hat{\eta}, \hat{\beta}\}$ via Equation (3). Subsequently, an arbitrary number m of new bootstrap samples, $\{im_{f,1k}^*, im_{f,2k}^*,.., im_{f,nk}^*\}$, $k = \{1,..,m\}$, is extracted from the reference distribution defined by $\{\hat{\eta}, \hat{\beta}\}$, with each new sample being of size n. Then, for the k-th out of m bootstrap samples, a new fragility function (Figure 5a) is evaluated via Equation (3), having parameters $\{\hat{\eta}_k^*, \hat{\beta}_k^*\}$, and the bootstrap replication of the failure rate $\hat{\lambda}_{f,k}^*$ is computed using these parameters and Equation (1). Finally, as in the resampling process, the mean and the variance of the failure rate estimators are calculated via Equation (11).

Figure 5: Reference fragility functions (black curves) and 200 bootstrap extractions of lognormal fragility (a) and empirical fragility (b).

Parametric bootstrap for EDP-based fragility

In the EDP-based case, the reference structural fragility parameters $\{\hat{\eta}, \hat{\beta}\}$ are obtained from the available responses via the binomial maximum likelihood of Equation (5). Then, at the j-th stripe, corresponding to $IM = im_j$, $j = \{1,..,u\}$, a number of m bootstrap samples is extracted from the binomial distribution with parameter equal to $p_j = \Phi\left\{\left[\ln\left(im_j\right) - \hat{\eta}\right] / \hat{\beta}\right\}$ (i.e., the parameter of binomial distribution is the probability of failure). Each sample consists of n Bernoulli trials, resulting in $q_{j,k}^{*}$ failures and $n - q_{j,k}^{*}$ survivals of the structure at the j-th stripe of the k-th bootstrap sample. Subsequently, new lognormal parameters $\{\hat{\eta}_k^{*}, \hat{\beta}_k^{*}\}$ are obtained from the $q_{j,k}^{*}$ failures, via Equation (5). It is assumed that, during the bootstrap replications, the maximum likelihood estimate may run into numerical problems for a number of m_o bootstrap samples, out of a total m. With this assumption in mind, the bootstrap replication of failure rate, $\hat{\lambda}_{f,k}^{*}$, is computed via Equation (1) and then the mean and variance of the estimator can be evaluated according to Equation (14):

$$\begin{cases} E\left[\hat{\lambda}_f\right] = \left(m - m_o\right)^{-1} \cdot \sum_{k=1}^{m-m_o} \hat{\lambda}_{f,k}^{*} \\ VAR\left[\hat{\lambda}_f\right] = \left(m - m_o - 1\right)^{-1} \cdot \sum_{k=1}^{m-m_o} \left(\hat{\lambda}_{f,k}^{*} - E\left[\hat{\lambda}_f\right]\right)^2 \end{cases}, \tag{13}$$

which only differs from Equation (11) in the fact that the simulation-based statistics are calculated using a number of $\left(m - m_o\right)$ bootstrap replications of the failure rate; i.e., only those that did not encounter numerical issues.

Delta method

An alternative method for evaluating the mean and variance of $\hat{\lambda}_f$, besides the bootstrap and the properties of the Gaussian function, is the delta method. The delta method uses a Taylor series expansion to approximate the expectation and variance of a RV and has been applied in the context of Cornell's seismic reliability method in [12]. The latter can be implemented using output from cloud analysis, which entails a set of n ground motion records with variable intensities and the corresponding sample of EDP responses. By performing linear regression of $\ln\left(EDP\right)$ against $\ln\left(IM\right)$ and assuming that the logarithm of the hazard curve, $\ln\left(\lambda_{im}\right)$, can be approximately considered locally linear, the annual failure rate $\hat{\lambda}_f$ can be estimated in closed-form, via Equation (15):

$$\hat{\lambda}_f \approx k_0 \cdot \left(\frac{edp_f}{\hat{a}}\right)^{-\frac{k}{b}} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\left(\hat{\beta}_D^2 + \beta_C^2\right)} \qquad , \tag{14}$$

where k_0 and k are, respectively, the slope and intercept of the $\ln(\lambda_{im})$ curve linearized around the IM corresponding to edp_f, $\hat{a}$ and $\hat{b}$ are the slope and intercept from the linear regression of $\ln(EDP)$ against $\ln(IM)$, $\hat{\beta}_D$ is the standard deviation of the residuals of $\ln(EDP)$ given $\ln(IM)$, which is estimated from the regression and β_C is the logarithmic standard deviation of edp_f, which is assumed to follow a lognormal distribution. In this context, the statistics of $\hat{\lambda}_f$ can be approximated via Taylor series expansion, according to Equation (16):

$$\begin{cases} E\left[\hat{\lambda}_f\right] = \hat{\lambda}_f + \frac{1}{2} \cdot VAR[\hat{a}] \cdot \frac{\partial^2 \hat{\lambda}_f}{\partial \hat{a}^2} + \frac{1}{2} \cdot VAR[\hat{b}] \cdot \frac{\partial^2 \hat{\lambda}_f}{\partial \hat{b}^2} + \frac{1}{2} \cdot VAR\left[\hat{\beta}_D^2\right] \cdot \frac{\partial^2 \hat{\lambda}_f}{\partial\left(\hat{\beta}_D^2\right)^2} + COV\left[\hat{a},\hat{b}\right] \cdot \frac{\partial^2 \hat{\lambda}_f}{\partial \hat{a} \cdot \partial \hat{b}} \\[4mm] VAR\left[\hat{\lambda}_f\right] = VAR[\hat{a}] \cdot \left(\frac{\partial \hat{\lambda}_f}{\partial \hat{a}}\right)^2 + VAR[\hat{b}] \cdot \left(\frac{\partial \hat{\lambda}_f}{\partial \hat{b}}\right)^2 + VAR\left[\hat{\beta}_D^2\right] \cdot \left[\frac{\partial \hat{\lambda}_f}{\partial\left(\hat{\beta}_D^2\right)}\right]^2 + 2 \cdot COV\left[\hat{a},\hat{b}\right] \cdot \frac{\partial \hat{\lambda}_f}{\partial \hat{a}} \cdot \frac{\partial \hat{\lambda}_f}{\partial \hat{b}} \end{cases} , \tag{15}$$

where the variances and covariance, the latter denoted by the operator $COV[\cdot]$, are known from linear regression theory, while the derivatives of $\hat{\lambda}_f$ can be obtained from Equation (14), but are also given in [12]. The delta method can also be applied for the failure rate estimator, under the assumption of lognormal fragility. In this case $\hat{\lambda}_f$ is regarded as a function of the fragility parameters $\{\hat{\eta},\hat{\beta}\}$, which was given previously as Equation (10). This function can undergo a Taylor series expansion analogous to the one given by Equation (15), which can be found in [12] along with the necessary derivatives of $\hat{\lambda}_f$. The advantage of the delta method, over the other procedures implemented in R2R-EU, is that the expressions such as those of Equation (15) need only be evaluated analytically once, after which the statistics of $\hat{\lambda}_f$ can be obtained with less computational effort with respect to the bootstrap.

Operational Outline

R2R-EU runs behind a Mathworks MATLAB®-based graphical user interface (GUI) which implements all of the methods illustrated in the previous section for evaluating a fragility model and for quantification of estimation uncertainty in the fragility parameters (where applicable) and in the failure rate. A flowchart of R2R-EU's operation is shown in Figure 6, where the primary operations of the software are categorized under the data input modules and those that produce and output the results.
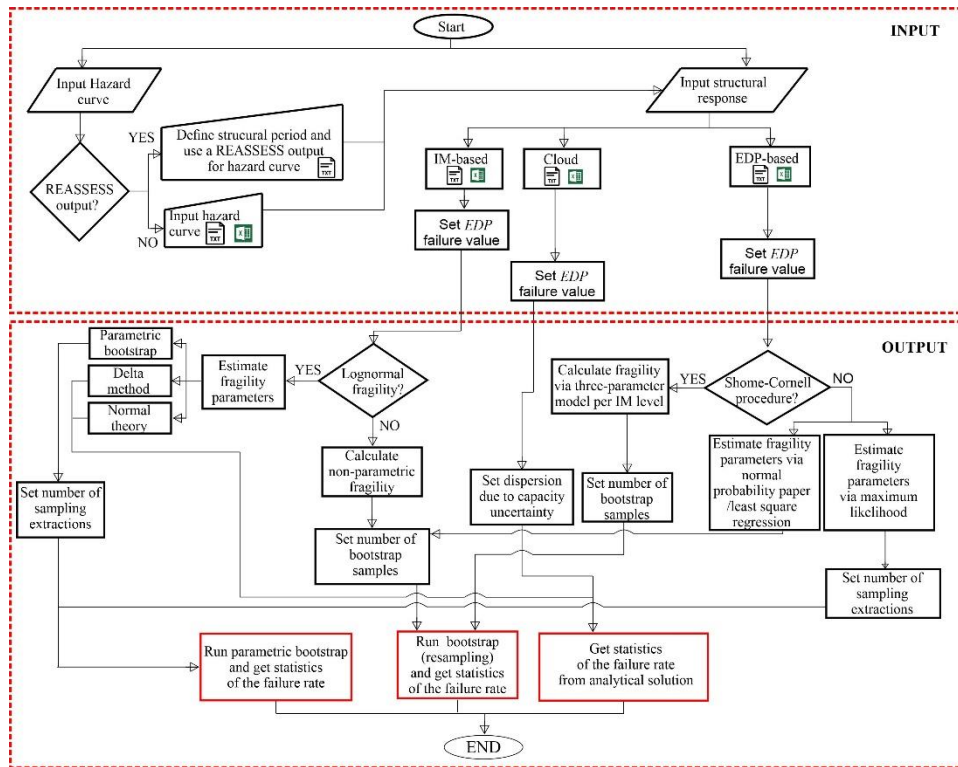

Figure 6: R2R-EU flowchart

Loading input data

First step when starting a new session in R2R-EU is for the user to provide two types of input: a hazard curve and structural response data needed to fit fragility functions. The tool offers two possibilities for data input: using two separate plain ASCII text (.txt) files (one for the hazard curve and one for structural response) or filling in the necessary values for both into an Excel spreadsheet template, with the latter option being activated by checking the box "Use Excel" (Figure 7).

Figure 7: Input Hazard curve and structural response

When using text files, one simply has to enter the file path and file name in the appropriate space provided, or press *Browse* and select a text file from its location on the disk.

For the **hazard curve**, the text file should be formatted into columns of equal length (separated by one space or more), with the first column representing values of some scalar seismic Intensity Measure (IM) and the second one representing the rate of exceedance of the IM values (header lines in the input file, as in the following example, are allowed but not necessary). The following example is extracted from the example file "Hazard.txt" contained in the folder "Example_Input_files".

IM	Rate of exceedance
0.00870	0.0458740
0.10037	0.0006836
0.20186	0.0000990
0.30564	0.0000265
0.50502	0.0000044

In cases when an output text file produced using the software REASSESS [14] is available, a hazard curve can be read directly from that file format by checking the box labeled "*REASSESS output*". In that case the user is also expected to fill in the box labeled "*Structural period for reading Sa(T) hazard curve*", since REASSES output files may contain hazard curves at different vibration periods (an example is stored in the folder "Example_Input_files", named "Analysis_Output_Hazard_Curves_site_1_Vs30_270_2.txt"). R2R-EU will load the hazard curve at the closest period to the desired value available in the input file.

In order to fit fragility functions to structural analysis data, the user has the possibility to choose between three different approaches, each requiring a somewhat different input format when using a text file: IM-based, EDP-based and Cloud analysis [15]. In each case, the corresponding box, labeled "IM" (intensity measure), "EDP" (engineering demand parameter) or "Cloud" respectively, should be checked prior to selecting the input file path and name (Figure 7).

 For the **IM-based** approach, the text file should be either a column or a space-separated line of numerical values, representing the IM value to which each record was scaled to cause the structure to fail. There should

be no headers or plain text characters in the file. An example format is given below (extracted from the example file "IM_based.txt" contained in the folder "Example_Input_files"), where numerical values represent scaled records' spectral acceleration values (in units of g), that caused the exceedance of some performance limit state:

0.48045   0.36675   0.28685   0.51613   0.56279   0.34842

In cases where incremental dynamic analysis (IDA) has been performed but only a fraction of the records used have been scaled to a level that causes the structure to fail, the user should also check the box labeled "**Truncated IDA**". This will enable the user to fill-in the box labeled *"Total number of records"* in order to account that IM-based input file is missing some values of failure intensities.

For the **EDP-based** approach, the input file should be formatted as a series of rows, containing numerical values separated by spaces. In each row, the first element should always contain a fixed IM level for which all the EDP responses following were obtained. All rows need not have the same number of data. In cases where some of the rows contain less elements than others, the difference will be assumed to correspond to non-convergent analyses signifying collapse and will therefore count as cases where the structure has failed.

An example of the appropriate text file formatting for the EDP-based approach (full example is stored in the folder "Example_Input_files", named "EDP_based.txt") is shown below, where the first number in each row represents spectral acceleration (units of g) that characterizes all records and the following numbers indicate maximum interstorey drift ratio of a frame building (header lines are allowed but not necessary):

IM   EDP

0.2   0.01715 0.01585 0.01923 0.01832 0.01743 0.01932

0.3   0.01888 0.02301 0.03237 0.02267 0.02087

0.4   0.02310 0.03778 0.04703 0.02537

0.5   0.03298 0.02936 0.02834

0.6   0.04422 0.03523

For **Cloud** analysis the text file should be formatted into columns of equal length (separated by one space or more), with the first column representing various IM levels and the second one representing the structural response, in terms of EDP, corresponding to that IM level.

An example of the appropriate text file formatting for the Cloud method is shown below (full example is stored in the folder "Example_Input_files", named "Cloud.txt"), where the first number in each row represents spectral acceleration (units of g) of each record and the following the number next to it indicates maximum interstorey drift ratio of a frame building (header lines are allowed but not necessary):

IM   EDP

0.05096   0.00581

0.01970   0.00231

0.09343   0.01005

0.17634   0.01664

0.08673   0.00934

In cases where the user needs to account for records that fail numerical convergence across all EDP levels considered (assumed to correspond to collapse cases), one has to fill-in the box labeled "Total number of records". This allows the program to derive the number of non-convergent runs as the difference between this number and the maximum number of responses present in the input file.

This software tool can also be used to integrate site-specific hazard with a lognormal fragility function of known parameters directly. This can be done by checking the box labeled "Override fragility" and pushing "Read", then proceeding to input the lognormal fragility parameters manually in the dialogue window that appears. (Figure 8)
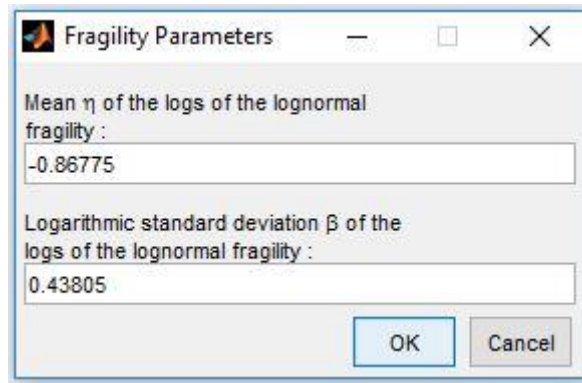
Figure 8: Input Fragility parameters

As previously mentioned, the tool also offers the possibility to use an **Excel spreadsheet template** for data input, instead of a text file, by checking the box labeled *"Use Excel"*. In this case the user has two possibilities (Figure 9): to fill-in the blank cells of the spreadsheet template that R2R-EU opens or to load previously compiled file(s). The latter possibility allows the user to select the **multiple input files to be analyzed** option. After the template has been filled-out (or the precompiled files have been selected), the "*Read*" button has to be pressed for the data to be loaded.
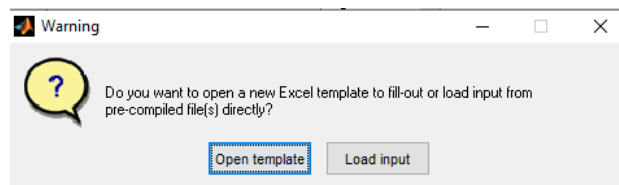
Figure 9: Message appearing when the box labeled "Use Excel" is selected

If the user chooses to fill-out the template spreadsheet, an Excel template will open. This comprises four sheets (Figure 10). The file "*Example_R2R_EU.xlsx*", which is stored in the installation folder by default, contains a complete example of data input via Excel environment.

Figure 10: Input Hazard curve by spreadsheet

The first sheet is labeled "Hazard" and has to be necessarily filled-in: The first column should contain IM values and the second column the corresponding rate of exceedance. Only one of the remaining three sheets

(labeled "IM_based", "EDP_based" and "Cloud") has to be compiled, depending on the chosen type of analysis.

In the sheet "IM_based" (Figure 11), each box in column A has to be filled in with the IM value to which each record was scaled to cause the structure to fail. The user has to set number of records (in the dedicated cell), while the box labeled "Number not collapsed" has to be equal to the difference between number of records and the length of the IM stripe.

In the sheet "EDP_based" (Figure 12), the user has to fill in row 3 with the IM values. The cells from row 5 and column B onwards should contain the EDP responses corresponding to the records exhibiting the IM level in the same column. Furthermore the user has to fill in the cell "Number of records" with the total number of records used in the analysis for each stripe.

In the sheet "Cloud" (Figure 13**Errore. L'origine riferimento non è stata trovata.**) the first column should contain IM values and the second one the structural responses corresponding to each IM value.

	A	B	C	D	E
1	IM		Number of records:		30
2	0.480452				
3	0.36676		Number not collapsed		0
4	0.286855				
5	0.516136		Intensity measure		Sa (0.5)
6	0.562797				
7	0.348428				
8	0.360125				
9	0.41944				
10	0.338318				

Figure 11: Input IM stripe by spreadsheet

	A	B	C	D	E	F	G	H	I	J	K	L
1						Number of records:		30		Intensity measure		Sa (0.5)
2												
3	IM	0.2	0.3	0.4	0.5	0.6						
4	EDP											
5	1	0.017154	0.018888	0.023108	0.032987	0.044223						
6	2	0.015856	0.023016	0.037787	0.029365	0.035237						
7	3	0.019233	0.032379	0.047038	0.028343	0.033071						
8	4	0.018329	0.022678	0.025376	0.03645	0.037946						
9	5	0.017431	0.020876	0.025806	0.043304	0.073268						
10	6	0.019323	0.027137	0.033002	0.042304	0.047243						
11	7	0.018587	0.023031	0.039578	0.03904	0.05223						
12	8	0.01759	0.024098	0.028482	0.025884	0.028427						
13	9	0.017484	0.026465	0.035985	0.023768	0.024751						

Figure 12: Input EDP stripes by spreadsheet

Figure 13: Input Cloud data by spreadsheet

As a special case of EDP-based data input, two sheets named "RINTC_x" and "RINTC_y" (Figure 14) are provided for users that wish to import standard output files used in the RINTC project [16]. The two sheets refer to response in two principal directions, "x" and "y". The R2R-EU tool internally combines the responses in the two directions to create a single EDP matrix in terms of maximum demand over capacity values (demand from the cells from row 3, column B to row 22, column K; capacity read from the cell at row 2, column L). In this input format, in the case of results that can be interpreted as collapse (e.g. lack of convergence or unrealistic response values due to numerical instability), this should be indicated **by filling in a value of -1** in the corresponding cell (Figure 14). In the presence of RINTC-type input, when the cells in the excel sheet "Hazard" remain blank, R2R-EU will automatically read the IM levels from the sheet "RINTC_x" (from line 2, column B to line 2 column K) and associate the exceedance rates calculated as the reciprocals of the return periods on the same sheet (from line 1, column B to line 1, column K).

	A	B Tr = 10 anni	C Tr = 50 anni	D Tr = 100 anni	E Tr = 250 anni	F Tr = 500 anni	G Tr = 1000 anni	H Tr = 2500 anni	I Tr = 5000 anni	J Tr = 10000 anni	K Tr = 100000 anni	L EDP soglia EDP stato limite
1												
2	Analisi	0.128	0.267	0.425	0.589	0.784	1.039	1.397	1.801	2.512	4.456	0.632
3	1	0.0007	0.0014	0.0018	0.0029	0.0129	0.0072	0.0070	0.0677	0.0184	0.4928	Intensity measure
4	2	0.0057	0.0104	0.0096	0.0237	0.0171	0.0169	0.0752	0.3999	-1.0000	-1.0000	Sa(0.5)
5	3	0.0012	0.0048	0.0171	0.0156	0.0040	0.0247	0.0903	0.0128	0.1107	0.1410	
6	4	0.0047	0.0111	0.0028	0.0251	0.0364	0.0455	0.0345	0.0865	0.1022	0.0757	
7	5	0.0044	0.0057	0.0127	0.0131	0.0367	0.0960	0.2242	0.0893	0.3789	-1.0000	
8	6	0.0034	0.0081	0.0075	0.0121	0.0152	0.0375	0.0805	0.0739	0.2296	0.1920	
9	7	0.0024	0.0082	0.0119	0.0264	0.0336	0.1247	0.0668	0.0579	0.1421	0.2306	
10	8	0.0050	0.0093	0.0157	0.0133	0.0157	0.0369	0.0660	0.0928	0.1132	0.1983	
11	9	0.0088	0.0075	0.0150	0.0108	0.0404	0.0376	0.0430	0.0980	0.0987	0.2710	
12	10	0.0023	0.0135	0.0200	0.0309	0.0294	0.0545	0.0496	0.0359	0.2278	0.2823	
13	11	0.0014	0.0099	0.0101	0.0138	0.0250	0.0283	0.0721	0.0502	0.2757	0.0964	
14	12	0.0043	0.0077	0.0176	0.0256	0.0228	0.0450	0.0386	0.0514	0.0518	0.1925	
15	13	0.0041	0.0058	0.0068	0.0097	0.0309	0.0235	0.0306	0.1143	0.1113	0.1328	
16	14	0.0023	0.0035	0.0079	0.0133	0.0176	0.0258	0.0711	0.1493	0.0749	0.7326	
17	15	0.0031	0.0050	0.0058	0.0071	0.0441	0.0447	0.0833	0.0883	0.0777	0.8212	
18	16	0.0036	0.0089	0.0147	0.0238	0.0376	0.0433	0.1512	0.0536	0.2800	0.9224	
19	17	0.0055	0.0131	0.0233	0.0100	0.0570	0.0316	0.0833	0.1297	0.2841	0.5526	
20	18	0.0056	0.0106	0.0054	0.0234	0.0321	0.0314	0.0598	0.0937	0.0722	0.6270	
21	19	0.0052	0.0034	0.0150	0.0372	0.0108	0.0487	0.0656	0.0865	0.1600	0.1467	
22	20	0.0016	0.0099	0.0060	0.0147	0.0474	0.0706	0.0349	0.1388	0.5420	0.5688	
23		AQ_SuoloC_Sa	AQ_SuoloC_Sa	AQ_SuoloC_Sa	AQ_SuoloC_Sa	AQ_SuoloC_Sa	AQ_SuoloC_Sa	AQ_SuoloC_Sa	AQ_SuoloC_Sa	AQ_SuoloC_Sa(0	AQ_SuoloC_Sa(0,1	Nome Striscia
24												
25		20	20	20	20	20	20	20	20	19	15	Num rec no coll.
26		0	0	0	0	0	0	0	0	0	3	Num rec coll. EDP
27		0	0	0	0	0	0	0	0	1	2	Num rec coll. inst.
28		20	20	20	20	20	20	20	20	20	20	Somma num rec

Figure 14: Input EDP stripes by spreadsheet in RINTC output format

For each type of analysis, If the cell "Intensity measure" has been filled, the corresponding string is going to be print instead of the generic "IM" in the figures showing structural response and fragility curve.

Running Analysis

After completing the data input phase and selecting the relevant type of analysis from the options available in the lower-left angle toolbox (labeled "Analysis Data"), analysis is launched by pressing the button labeled "Run analysis". Analysis options such as number of bootstrap extractions/simulations to be performed (see

[15]) can be adjusted by overwriting the corresponding boxes under "Analysis Data". For all types of analysis, the rate of failure resulting from integration of the fitted fragility function with the hazard will appear in the red-framed box on the top of the GUI. In Figure 15-Figure 16, a completed analysis performed under the IM-based/EDP-based approach using non-parametric/parametric fragility, is shown. For the EDP-based approach, if a method using the modelled fractiles from equation (6) is used, the user has the possibility to choose if the probability of collapse has to be empirical (as shown in equation (6)) or evaluated by means the logistic regression in equation (7). Table 2 sums up the type of results appearing in each of the four numbered output windows for every type of analysis (indicated as 1,2,3 and 4 in Figure 15).
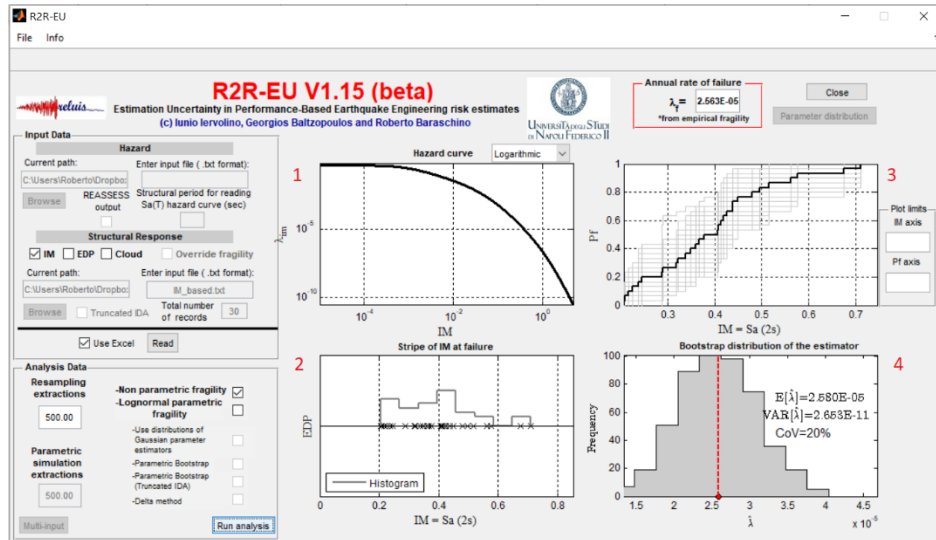
Figure 15: Snapshot of the GUI after the conclusion of IM-based analysis
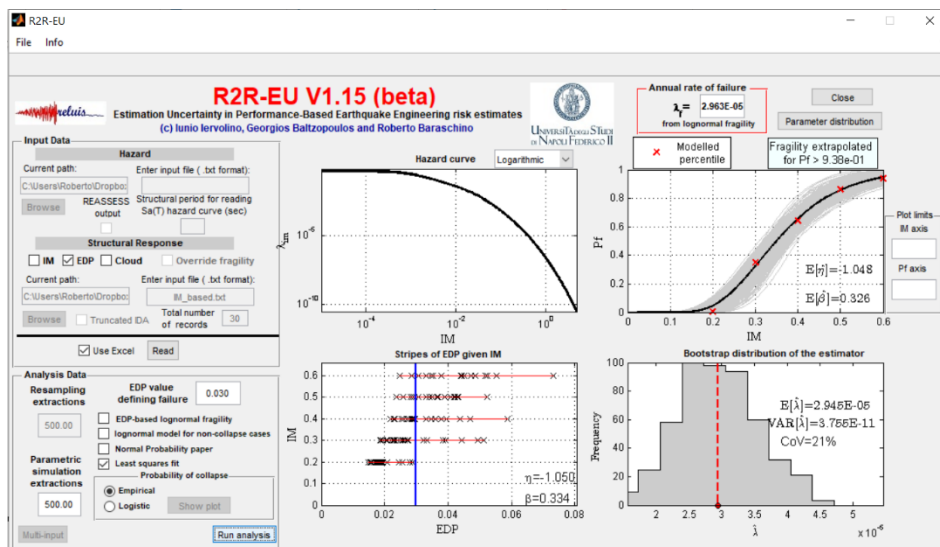
Figure 16: Snapshot of the GUI after the conclusion of EDP-based analysis.

Analysis	Case	Results appearing in window no. 1	Results appearing in window no. 2	Results appearing in window no. 3	Results appearing in window no. 4
IM	Non parametric fragility	User-defined hazard curve	Histogram of IM at failure	Empirical distributions from resampling of the empirical fragility function	Bootstrap distribution of the failure rate estimator
	Gaussian parameter estimators		Histogram of IM at failure, fitted lognormal fragility model and its parameters	Lognormal fragility curve and expected values of the parameter estimators from theoretical distributions	Failure rate estimator statistics
	Parametric Bootstrap		Histogram of IM at failure, fitted lognormal fragility model and its parameters	Parametric bootstrap realizations of lognormal fragility function and resulting expected values of the fragility parameter estimators	Bootstrap distribution of the failure rate estimator
	Parametric Bootstrap (Truncated IDA)		Histogram of IM at failure, fitted lognormal fragility model and its parameters	Parametric bootstrap realizations of lognormal fragility function and resulting expected values of the fragility parameter estimators	Bootstrap distribution of the failure rate estimator
	Delta method		Histogram of IM at failure, fitted lognormal fragility model and its parameters	Lognormal fragility curve and expected values of the parameter estimators from theoretical distributions	Failure rate estimator statistics

Table 2: Cases of output per type of analysis (IM-based)

Analysis	Case	Results appearing in window no. 1	Results appearing in window no. 2	Results appearing in window no. 3	Results appearing in window no. 4
EDP	Lognormal fragility	User-defined hazard curve	Stripes of EDP given IM, EDP defining failure	Lognormal fragility curve, bootstrap realizations and resulting expected values of the fragility parameter estimators	Bootstrap distribution of the failure rate estimator
	Normal Probability paper/ least squares fit			Lognormal fragility curve, bootstrap realizations and resulting expected values of the fragility parameter estimators	

Table 3: Cases of output per type of analysis (EDP-based)

If multiple file have been loaded, pushing the button *"Run Analysis"* will launch the same type of analysis for all files, to be run consecutively. The user may instead choose to run analysis for only part of the input files by pressing the button *"Multi-input"* and selecting the cases of interest.

The pop-up menu in the lower left part serves for switching visualization from one input case to another. For example, in the case shown in Figure 17, if the user selects the option *"Run analysis"* without making any prior selections via the *"Multi-input"* button, the pre-selected type of analysis, i.e., *"Parametric bootstrap"* will be launched for all cases previously loaded.
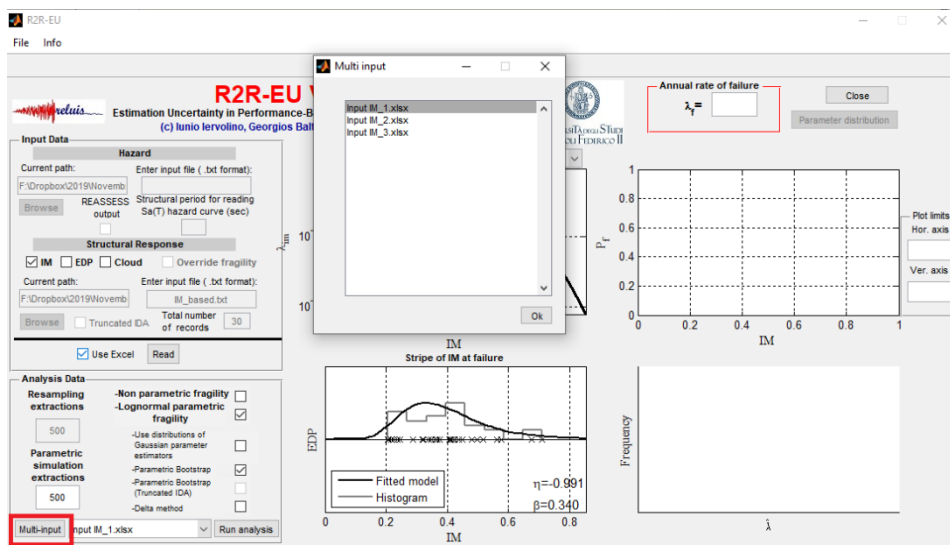
Figure 17: Multi-input selection

Saving a project and analysis output

The user can save/load a project at the end of the analysis stage from the File menu, found at the top of the GUI. In the folder "Saved results" (of the installation directory) three example projects are already stored, one for every type of analysis (filenames project_IM.mat, project_EDP.mat and project_Cloud.mat). To clear all results from a project and start a new analysis, the user can use the command "New project" from the File menu. The analysis results may be exported to a text or .mat file by choosing Export Output (also from the File menu). An example of the output file created can be seen in Figure 18

If analysis has been run for multiple input files, a separate output file will be generated for each one (name "Filename_{number of input file}.txt")
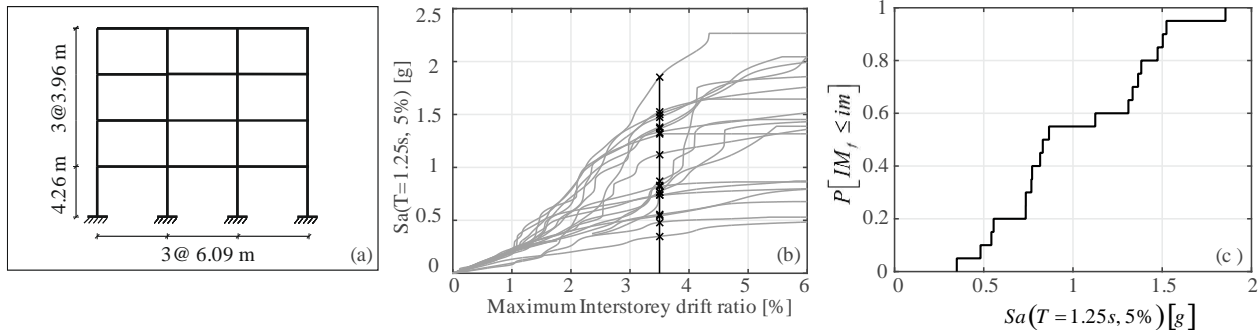

**Figure 18: Analysis output in text format**

Examples

IM-based approach

In this section an application of R2R-EU is presented, using as case study structure a four-story, plane, code-conforming, steel perimeter moment resisting frame designed to ASCE-SEI 7-05 criteria and described in the NIST GCR 10-917-8 report. (Figure 19a).

Figure 19: perimeter frame for the case study of IM-based fragility (a); IDA curves for the case study frame, from twenty records, in terms of maximum interstorey drift ratio and 5% damped spectral acceleration at the structure's first-mode period (b), with crosses marking the *IM_f* vertical stripe at failure defined by IDR>3.5 %; non-parametric fragility corresponding to the vertical *IM_f* stripe shown (c).

The structure is ideally located at a site near the town of Amatrice (central Italy; lat. 42.53°, lon. 13.29°), for which the hazard curve (Figure 20), in terms of five-percent-damped spectral acceleration at the frame's first-mode vibration period $Sa(T=1.25\,\text{s},5\%)$ is obtained using the software REASSESS, considering the seismic source model from . IDA is performed for the structure, using a set of fifty records (selected from NESS1, a dataset of the *Engineering Strong Motion database* [17]), which are scaled upwards until side-sway collapse (Figure 19b). By considering a generic limit state, which is nominally exceeded when a maximum interstorey drift ratio (IDR) above 3.5% is recorded, the R2R-EU software is used for the quantification of the failure rate's estimation uncertainty. To this end, the example uses two of the available strategies: the bootstrap for non-parametric IM-based fragility (shown in Figure 19c) and the delta method for lognormal fragility. The exercise is repeated in two versions: the first only uses a randomly selected subset of twenty-out-of-fifty IDA curves, while the entire set of fifty is used on the second go.
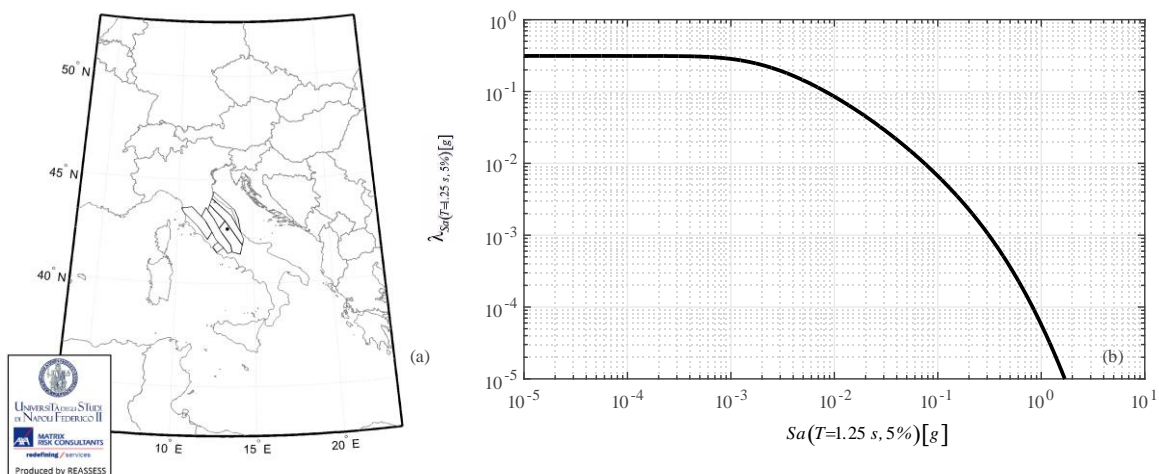
Figure 20: Seismogenic zones considered in the hazard analysis in REASSESS and location of the site (black dot) (a), resulting hazard curve (b).

For the application of the bootstrap resampling method, first the vectors composed by twenty (and later fifty) IM_f values (crosses in Figure 19b) are used to build the empirical fragility curves. In this case, five-hundred bootstrap extractions of the failure rate are requested from R2R-EU, which leads to calculating the mean and variance of the estimator, according to the methodology outlined in paragraph 0. These statistics are also

calculated by means of the delta method and the whole process is repeated using the structural response results from all fifty records; the results provided by R2R-EU are summarized in Table 4, where the coefficient of variation reported in the last column is calculated as $CoV_{\hat{\lambda}_f} = \sqrt{VAR\left[\hat{\lambda}_f\right]}\Big/E\left[\hat{\lambda}_f\right]$. The drop in the dispersion of the estimator with the increase in the number of records used, is evident across both inference methods employed.

Method	*Number of records*	$\hat{\lambda}_f$ [events/year]	$E\left[\hat{\lambda}_f\right]$ [events/year]	$VAR\left[\hat{\lambda}_f\right]$ [(events/year)²]	$CoV_{\hat{\lambda}_f}$
Non-parametric bootstrap (IM-based)	20	$1.53\cdot10^{-4}$	$1.59\cdot10^{-4}$	$2.34\cdot10^{-9}$	30%
	50	$1.52\cdot10^{-4}$	$1.53\cdot10^{-4}$	$4.32\cdot10^{-10}$	14%
Delta method	20	$1.36\cdot10^{-4}$	$1.41\cdot10^{-4}$	$1.82\cdot10^{-9}$	30%
	50	$1.48\cdot10^{-4}$	$1.48\cdot10^{-4}$	$4.68\cdot10^{-10}$	15%

Table 4: Statistics of the failure rate estimator evaluated using samples of either twenty or fifty records to obtain structural response via IDA.

EDP-based approach: applications from the RINTC project

This application considers six of the structures that were designed, modelled and analysed within the activities of the RINTC project (in italian: *Rischio Implicito delle strutture progettate secondo le Norme Tecniche per le Costruzioni* – implicit seismic risk of code-conforming Italian buildings), which is a research project carried out by a consortium of Italian universities (ReLUIS) for the Department of Civil Protection, with the objective of analytically assessing the intrinsic seismic reliability of new code-conforming structures in Italy [16,18]. Within the framework of that project, MSA was used to estimate the seismic reliability for buildings of varying typology and configuration, designed for several Italian sites in order to cover a wide range of seismic hazard levels. For the purposes of this illustrative application of the R2R-EU software, the buildings considered were designed for the site of L'Aquila under soil conditions of *type C*, according to Eurocode 8 classification [19], and belong to the following typologies: unreinforced masonry residential buildings (URM), cast-in-place reinforced concrete residential buildings (RC) and steel industrial buildings (STEEL). The three-dimensional numerical model of each building was subjected to MSA using ten stripes, each containing twenty records of *hazard-consistent*, bi-directional, ground motion. The shaking intensity, characteristic of each stripe, was selected to correspond to a specific return period (T_r), ranging from 10 years to 10^5 years, and the analysis results were combined with the site-specific hazard to evaluate λ_f for the collapse limit state (among others) [16]. The six buildings used for the application shown here (two of each aforementioned category STEEL, URM, RC) are briefly described below:

STEEL 1: single-storey steel industrial building consisting of 20 m span-length moment-resisting portal frames in the transverse direction, which are repeating every 6 m in the longitudinal sense and are braced via concentric braces at the outermost spans;

STEEL 2: steel structure similar to STEEL 1 only somewhat larger, with axial distance between portal frames and braced span length equal to 8 m;

URM 1: Regular two-story unreinforced masonry structure (configuration type "C2" [20]);

URM 2: Regular in elevation and irregular in plan unreinforced masonry structure (configuration type "I1" [20])

RC 1: cast-in-place reinforced concrete structure consisting of a nine-story moment-resisting frame with masonry infills, coupled with a shear wall;

RC 2: cast-in-place reinforced concrete structure consisting of a nine-story moment-resisting and shear wall pilotis frame, i.e., designed for interruption of the masonry infills at the ground floor;

Details on the design, numerical modelling and non-linear dynamic analyses of these structures can be found in [21–23].

All three EDP-based procedures for MSA available in R2R-EU (i.e., one parametric and two resampling bootstrap processes, see Table 1) were implemented for these six structures. The dynamic analysis results that underlie the R2R-EU elaborations are presented in Figure 21, where structural responses per stripe are given in terms of demand-over-capacity ratios (D/C), these ratios being defined as the EDP values normalized by the edp_f that has been assigned to signify conventional collapse for each structure. This means that failure corresponds to D/C exceeding unit value, which has been marked on Figure 21 by a vertical dashed line; on the same figure, all converged-analysis cases with $D/C > 2$ have been incorporated along with non-convergent analyses into a single point per stripe labelled as *collapses*, that carries a label reporting the number of conventional collapse cases. The results provided by R2R-EU, corresponding to these analyses, in terms of point estimate $\hat{\lambda}_f$ and estimator statistics for the failure rate and fragility parameters (where applicable) are summarized in the Table 5.[1]
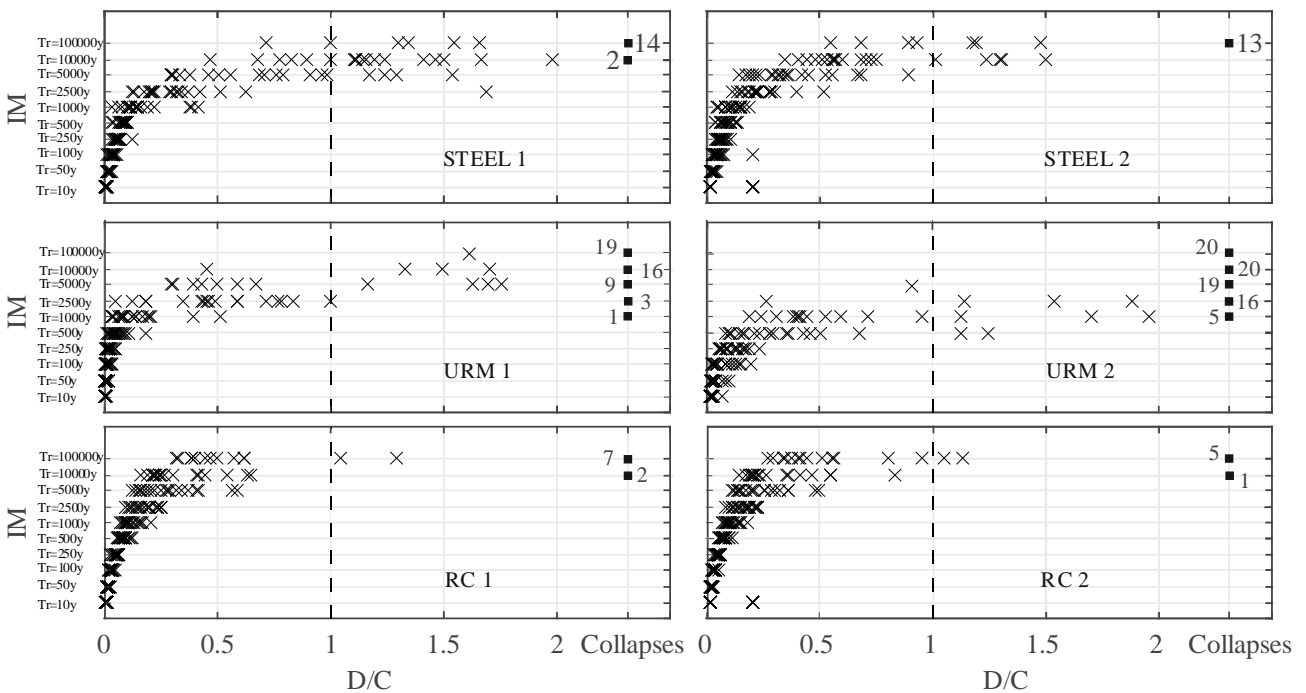
Figure 21: Structural response of the six considered buildings in terms of demand-over-capacity ratios D/C. Crosses represent response values from dynamic analysis that converged with $D/C \le 2$; squares are stand-ins used for indicating the number of conventional collapses at each stripe.

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[1] It should be noted that it has been RINTC practice to refrain from extending hazard calculations to intensities with $T_R > 10^5$ years, so as to avoid extrapolating semi-empirical seismological models, assuming that λ_f for collapse is at least 10^{-5} [16]; in the applications presented herein this condition is dropped and the failure rates are solely dependent on the structural responses available.

Lognormal fragility (maximum likelihood)

| Building	$\hat{\lambda}_f$ [events/year]	$(m - m_o)$	$E[\hat{\eta}]$	$VAR[\hat{\eta}]$	$E[\hat{\beta}]$	$VAR[\hat{\beta}]$	$E[\hat{\lambda}_f]$ [events/year]	$VAR[\hat{\lambda}_f]$ [(events/year)²]	$CoV_{\hat{\lambda}_f}$
STEEL 1	$2.07\cdot10^{-4}$	500	1.26	$7.26\cdot10^{-3}$	$4.51\cdot10^{-1}$	$6.72\cdot10^{-5}$	$2.08\cdot10^{-4}$	$1.13\cdot10^{-9}$	16%
STEEL 2	$9.65\cdot10^{-5}$	491	1.68	$8.59\cdot10^{-3}$	$3.78\cdot10^{-1}$	$5.02\cdot10^{-5}$	$9.65\cdot10^{-5}$	$1.81\cdot10^{-10}$	14%
URM 1	$3.81\cdot10^{-4}$	500	0.52	$1.79\cdot10^{-3}$	$2.35\cdot10^{-1}$	$2.14\cdot10^{-5}$	$3.81\cdot10^{-4}$	$2.44\cdot10^{-9}$	13%
URM 2	$1.19\cdot10^{-3}$	495	0.74	$2.04\cdot10^{-3}$	$2.28\cdot10^{-1}$	$1.39\cdot10^{-3}$	$1.19\cdot10^{-3}$	$2.28\cdot10^{-8}$	13%
RC 1	$5.08\cdot10^{-5}$	431	1.32	$2.51\cdot10^{-2}$	$4.98\cdot10^{-1}$	$1.35\cdot10^{-2}$	$5.39\cdot10^{-5}$	$1.62\cdot10^{-10}$	24%
RC 2	$3.66\cdot10^{-5}$	298	1.47	$6.85\cdot10^{-2}$	$5.37\cdot10^{-1}$	$2.38\cdot10^{-3}$	$4.16\cdot10^{-5}$	$1.09\cdot10^{-10}$	25%

Lognormal fragility (normal probability paper procedure)

Building	$\hat{\lambda}_f$ [events/year]	m	$E[\hat{\eta}]$	$VAR[\hat{\eta}]$	$E[\hat{\beta}]$	$VAR[\hat{\beta}]$	$E[\hat{\lambda}_f]$ [events/year]	$VAR[\hat{\lambda}_f]$ [(events/year)²]	$CoV_{\hat{\lambda}_f}$
STEEL 1	$1.42\cdot10^{-4}$	500	1.36	$2.17\cdot10^{-2}$	$2.06\cdot10^{-1}$	$1.13\cdot10^{-3}$	$1.40\cdot10^{-4}$	$7.84\cdot10^{-10}$	20%
STEEL 2	$1.06\cdot10^{-4}$	500	1.55	$2.26\cdot10^{-2}$	$2.39\cdot10^{-1}$	$1.36\cdot10^{-3}$	$1.08\cdot10^{-4}$	$4.30\cdot10^{-10}$	19%
URM 1	$3.10\cdot10^{-4}$	500	0.57	$2.89\cdot10^{-3}$	$1.78\cdot10^{-1}$	$6.03\cdot10^{-4}$	$3.10\cdot10^{-4}$	$1.51\cdot10^{-9}$	13%
URM 2	$8.09\cdot10^{-4}$	500	0.20	$3.95\cdot10^{-3}$	$1.95\cdot10^{-1}$	$1.65\cdot10^{-3}$	$8.33\cdot10^{-4}$	$2.68\cdot10^{-8}$	20%
RC 1	$9.98\cdot10^{-5}$	500	0.90	$1.38\cdot10^{-2}$	$1.57\cdot10^{-1}$	$3.03\cdot10^{-4}$	$9.15\cdot10^{-5}$	$1.20\cdot10^{-10}$	12%
RC 2	$6.15\cdot10^{-5}$	500	1.13	$2.50\cdot10^{-2}$	$1.85\cdot10^{-1}$	$6.87\cdot10^{-4}$	$6.44\cdot10^{-5}$	$4.70\cdot10^{-10}$	34%

Three-parameter model per intensity, Equation (6)

Building	$\hat{\lambda}_f$ [events/year]	m	$E[\hat{\lambda}_f]$ [events/year]	$VAR[\hat{\lambda}_f]$ [(events/year)²]	$CoV_{\hat{\lambda}_f}$
STEEL 1	$1.85\cdot10^{-4}$	500	$1.88\cdot10^{-4}$	$3.03\cdot10^{-10}$	9%
STEEL 2	$9.52\cdot10^{-5}$	500	$9.56\cdot10^{-5}$	$8.70\cdot10^{-11}$	10%
URM 1	$4.10\cdot10^{-4}$	500	$4.12\cdot10^{-4}$	$2.95\cdot10^{-9}$	13%
URM 2	$1.13\cdot10^{-3}$	500	$1.12\cdot10^{-3}$	$9.45\cdot10^{-9}$	9%
RC 1	$4.48\cdot10^{-5}$	500	$4.61\cdot10^{-5}$	$1.43\cdot10^{-10}$	26%
RC 2	$3.24\cdot10^{-5}$	500	$3.26\cdot10^{-5}$	$9.71\cdot10^{-11}$	30%

Table 5: Statistics of the estimator of the failure rate by using all three procedures implemented in R2R-EU for the MSA EDP-based approach.
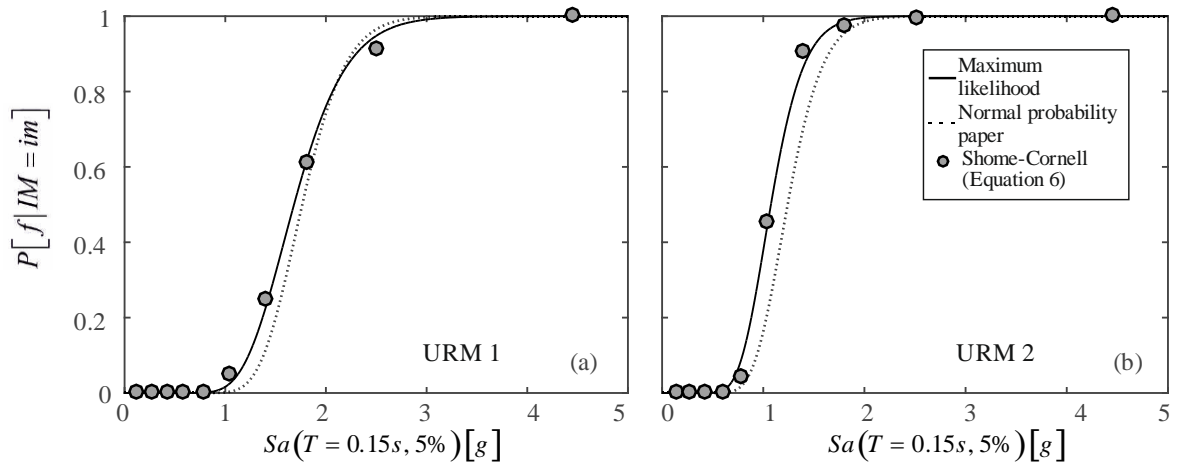
Figure 22: Fragility functions estimated by R2R-EU from the MSA results of the two masonry buildings: URM 1 (a) and URM 2 (b).

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